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ing that FP, PA' are now the displacements in time δt of the point F, $\left(A\omega_1 + A\frac{d\omega_1}{dt}.\delta t,\ 0,\ 0\right)$, &c.), the following as the resultant changes of the moment of momentum in time δt:

$$\left(A\frac{d\omega_1}{dt} - B\omega_2\omega_3 + C\omega_3\omega_2\right)\delta t \text{ along } OA$$

$$\left(A\omega_1\omega_3 + B\frac{d\omega_2}{dt} - C\omega_3\omega_1\right)\delta t \text{ along } OB$$

$$\left(-A\omega_1\omega_2 + B\omega_2\omega_1 + C\frac{d\omega_3}{dt}\right)\delta t \text{ along } OC.$$

The changes per unit time are therefore $A\frac{d\omega_1}{dt} - (B - C)\,\omega_2\omega_3$, &c., in the directions OA, OB, OC, respectively.

21st November, 1878.

IV.

Condition of a Straight Line Touching a Surface.

By James Loudon, *University College, Toronto, Canada.*

The following method of getting the condition of a straight line touching a surface I have used for some years, and expected to see it in Salmon's 3d edition. It was so short and unimportant that I never thought of publishing it. (See Salmon's *Geometry of Three Dimensions, Art.* 80.)

Let $u \equiv ax^2 + by^2 + cz^2 + dw^2 + 2fyz + 2gzx + 2hxy + 2lxw + 2myw + 2nzw = 0$, then $P \equiv \alpha x + \beta y + \gamma z + \delta w = 0$, $P' \equiv \alpha' x + \beta' y + \gamma' z + \delta' w = 0$ is a line touching $u = 0$. Therefore $P - \lambda P' = 0$ is, for some value of λ, the tangent plane at the point of contact (x', y', z', w'); *i. e.*

$(\alpha - \lambda\alpha')\,x + (\beta - \lambda\beta')\,y + (\gamma - \lambda\gamma')\,z + (\delta - \lambda\delta')\,w = 0$ and
$(ax' + hy' + gz' + lw')\,x + (hx' + by' + fz' + mw')\,y + (gx' + fy' + cz' + nw')\,z$
$+ (lx' + my' + nz' + dw')\,w = 0$ are identical. Therefore

$$\begin{array}{ll} ax' + hy' + gz' + lw' = k\,(\alpha - \lambda\alpha') & hx' + by' + fz' + mw' = k\,(\beta - \lambda\beta') \\ gx' + fy' + cz' + nw' = k\,(\gamma - \lambda\gamma') & lx' + my' + nz' + dw' = k\,(\delta - \lambda\delta') \\ \alpha x' + \beta y' + \gamma z' + \delta w' = 0 & \alpha' x' + \beta' y' + \gamma' z' + \delta' w' = 0. \end{array}$$

Eliminating x', y', z', w', $-k$, $k\lambda$, we get the required condition

$$\begin{vmatrix} a, & h, & g, & l, & \alpha, & \alpha' \\ h, & b, & f, & m, & \beta, & \beta' \\ g, & f, & c, & n, & \gamma, & \gamma' \\ l, & m, & n, & d, & \delta, & \delta' \\ \alpha, & \beta, & \gamma, & \delta, & 0, & 0 \\ \alpha', & \beta', & \gamma', & \delta', & 0, & 0 \end{vmatrix} = 0.$$

28th November, 1878.